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Heat Transfer from Continuously Moving Material in Channel Flow for Thermal Processing

B. H. Kang* and Y. Jaluria†
Rutgers University, New Brunswick, New Jersey 08903

The conjugate mixed convection and conduction transport that arises due to the continuous movement of a heated material in a parallel channel flow of a cooler fluid has been numerically investigated. The basic problem studied is of interest in several manufacturing processes, ranging from crystal growing and continuous casting to extrusion, hot rolling, wire drawing, and glass fiber drawing. A flat plate, or sheet, moving in a wide channel is considered in this article. A two-dimensional, steady circumstance, with laminar flow in the fluid, is assumed. The full, elliptic, governing equations are solved, employing finite difference techniques. Numerical results are obtained for various values of the important physical parameters such as plate speed U_s, the forced flow velocity in the channel U_∞, and temperature T_0 of the material upstream of the cooling region. The effect of the geometry, or configuration, of the system is also investigated. The results obtained indicate that, as expected, the effect of thermal buoyancy is more significant when the plate is moving vertically upward than when it is moving horizontally, and also when the forced flow velocity is small. A strong forced flow leads to a fairly uniform velocity distribution across much of the channel. The plate speed substantially affects the overall heat transfer rate. Although this work is focused on convective cooling of a heated moving material, the basic considerations and many of the results can easily be extended to a moving material which is being heated through convection, i.e., in a heat treatment process.

Nomenclature

C = specific heat of solid or fluid at constant pressure
d = half-thickness of the moving flat plate
Gr = Grashof number, $[g\beta(T_0 - T_\infty)d^3/\nu^2]$
g = magnitude of gravitational acceleration
H = channel width
h = surface convective heat transfer coefficient
K = thermal conductivity
Nu = local Nusselt number, defined in Eq. (12)
Pe = Peclet number, $U_s d/\alpha_s$
Pr = Prandtl number, ν/α_f
p = local pressure
Re = Reynolds number, $U_s d/\nu$
Ri = Richardson number, Gr/Re^2
T = local physical temperature
T_0 = specified uniform temperature of the moving solid material far upstream
t = physical time
U = dimensionless velocity component in x direction, u/U_s
U_{max} = maximum U velocity component at a given downstream location
U_s = physical speed of the plate
U_∞ = inlet physical velocity of the forced flow in the channel
u = velocity component in x direction
V = dimensionless velocity component in y direction, v/U
$\bar{V}$ = dimensionless velocity vector, $\bar{v}/U_s$
v = velocity component in y direction
$\bar{v}$ = velocity vector, $\bar{i}u + \bar{j}v$
X = dimensionless coordinate distance along the length of the plate, x/d
X_b = dimensionless x_b, x_b/d
x = coordinate distance along the length of the plate
x_b = upstream distance where $T = T_0$ is specified in the numerical scheme
Y = dimensionless coordinate distance, normal to the plate surface, y/d
y = coordinate distance, normal to the plate surface
α = thermal diffusivity, $K/\rho C$
β = coefficient of thermal expansion of the fluid
ε = convergence criterion for steady state
θ = dimensionless local temperature, $[(T - T_\infty)/(T_0 - T_\infty)]$
ν = kinematic viscosity of the fluid
ρ = density
τ = dimensionless time
Ψ = dimensionless stream function, $\psi/U_s d$
ψ = stream function
Ω = dimensionless vorticity, $\omega d/U_s$
ω = vorticity

Subscripts
f = fluid
s = solid, i.e., moving plate material
∞ = ambient medium

Introduction

A PROBLEM of substantial practical and fundamental interest is that of the heat transfer associated with a continuously moving material which loses energy by convection and radiation at the surface to the ambient fluid. This circumstance is of interest in a variety of manufacturing processes such as hot rolling, plastic extrusion, metal forming, glass fiber drawing, and continuous casting.[1–3] In most cases, the ambient fluid, far from the moving material, is essentially stationary, with the fluid flow being induced by the motion of the solid material, which is undergoing the given thermal process, and by thermal buoyancy. Then, the resulting flow and thermal fields are determined by these two mechanisms, buoyancy and material motion. However, in many practical

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*Graduate Assistant, Department of Mechanical and Aerospace Engineering; currently at Korea Institute of Science & Technology, Seoul, Republic of Korea.

†Professor, Department of Mechanical and Aerospace Engineering.

processes, the resulting flow does not provide adequate cooling rates, and an additional, externally induced, forced flow is employed to increase the heat transfer rate.[4] Usually, such a flow is generated in a channel or in a duct, so that the forced flow is aligned with the material motion. Other heat transfer mechanisms, such as boiling with the use of sprays and radiation, may also be employed for increasing the heat transfer rates.[5,6] Similar considerations arise if the material is being heated, i.e., in heat treatment and baking of continuously moving materials.

The main features of the two-dimensional flow induced by a long, continuously moving flat plate in a parallel channel flow of a given fluid are shown schematically in Fig. 1 for one of the two geometric configurations considered there. These involve a plate moving vertically or horizontally in the forced flow induced in the channel. The plate moves out of the slot in an extrusion die, or between the rollers in a hot rolling process, and proceeds downstream at constant speed U_s. The forced flow in the channel is employed to increase the thermal transport from the plate to the fluid. Due to the viscous drag exerted on the ambient fluid by the moving surface of the plate, the forced flow is accelerated near the plate if the plate speed is larger than the external flow, i.e., $U_s > U_\infty$, and the flow is retarded if $U_s < U_\infty$. The plate also exchanges thermal energy with the fluid and generates a buoyancy force in the flow. Therefore, the resulting flowfield in the channel is a consequence of the material motion, externally induced forced flow, and thermal buoyancy. Because of these effects, the transport process depends on the orientation and geometrical configuration for a given application.

Sakiadis[7] was the first investigator to analyze the boundary layer generated by a continuous sheet issuing from a slot into a quiescent ambient fluid. Detailed theoretical and experimental investigations of this problem have been carried by Tsou et al.,[8] Griffin and Thorne,[9] and Moutsoglou and Chen,[10] among others. In recent years, considerable work has also been done on the conjugate transport from a plate of finite thickness moving through an otherwise quiescent isothermal medium. Chida and Katto[11] computed the flow and temperature fields for the plate, employing boundary-layer approximations. Neglecting the effects of buoyancy, they obtained the downstream temperature variation. Experiments were also carried out with heated moving flat belts. A good agreement between the numerical and experimental results was obtained. Karwe and Jaluria[12,13] studied, numerically, the heat transfer from a plate, or a sheet, moving in quiescent, isothermal, ambient medium. A comparison of the results from the solution of the boundary-layer equations with those from the full elliptic equations was made. It was found that the nonboundary-layer effects are important near the slot and decay downstream, as expected. Karwe and Jaluria[14] also carried out an experimental study of a heated plate moving in quiescent water or air medium. A good agreement between the numerical and experimental results was found. Kang et al.[15] have numerically investigated the conjugate transport that arises due to the continuous movement of a heated plate in a quiescent medium. The transient effects were studied in detail. A recirculating region was found to arise in the flow at short time and to affect the local heat transfer substantially. With increasing time, this recirculation was swept downstream, leading to steady-state conditions upstream.

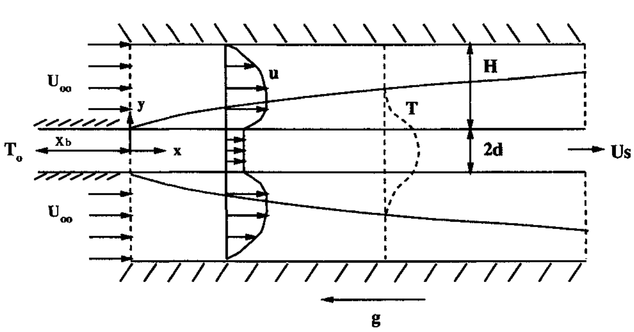

Fig. 1 Schematic of the velocity and temperature profiles in the conjugate problem for a continuously moving plate in a parallel channel flow, with the plate moving vertically upward.

In response to the cooling requirements for certain practical applications, such as those in materials processing, an additional forced flow may be employed to enhance the heat transfer rate from the heated plate to the fluid. Mixed convection from stationary surfaces has been studied extensively in the literature.[16] Forced air cooling of a moving fiber has been investigated experimentally and theoretically, with an assumed convective heat transfer coefficient at the fiber surface.[17] Not much work has been done on the convective heat transfer from heated plate moving in a channel with an externally induced forced flow of the cooling fluid.

The present study is directed at the conjugate transport from a continuously moving flat plate in a parallel channel flow, with the fluid flow in the same direction as the motion of the plate. Therefore, the two transport mechanisms, due to material motion and forced flow, are taken as aligned. For opposing forced flow, separation may arise, resulting in a decrease in the heat transfer rate. The effects of thermal buoyancy are included. The plate is assumed to be maintained at a given uniform temperature T_0 far upstream of the point where it emerges from a slot, representing an extrusion die, furnace outlet, etc. The velocity field in the flow and the temperature distributions in the solid, as well as in the flow, are studied in detail. Numerical calculations are carried out, assuming a two-dimensional steady circumstance, with laminar flow in the fluid. The laminar flow assumption is valid for small material speeds and forced flow velocities, such as those encountered in several forming processes. However, turbulent flow modeling will be necessary for many practical cases. The full, elliptic, equations that govern the heat transfer and the flow are solved, employing finite difference techniques, in order to study the effects of axial diffusion and nonboundary layer behavior in the fluid near the slot. These considerations are particularly important in materials processing due to the large temperature gradients in the solid material in this region and the effect of the temperature field on material properties.

The results obtained lead to a better understanding of the underlying physical processes and also provide inputs that may be used to design the relevant manufacturing system. This work differs from earlier investigations on this problem due to the inclusion of forced flow in the fluid, in addition to the flow generated by material motion and thermal buoyancy. This consideration brings in several important aspects in the analysis and in the physical trends obtained from the results. Also, the transport processes in a two-dimensional channel are investigated, instead of those in an extensive ambient medium.

Analysis

Consider the flow induced by a flat plate moving at a constant velocity U_s in a parallel channel flow with an inlet velocity of U_∞, as shown in Fig. 1. The transport in the solid material is coupled with that in the fluid through the boundary conditions, and the two have to be solved for simultaneously. The temperature of the plate is assumed to be at a uniform value of T_0 at an upstream distance x_b away from the origin, i.e., at $x = -x_b$. Basically, the condition of uniform temperature far upstream of the point of emergence is simulated, as that in a furnace or an oven. The effect of this upstream distance x_b on the thermal field was considered by Karwe and Jaluria.[13] They found significant effect of varying x_b, for small values of x_b. In the present study, X_b is fixed at 10, which was found numerically to be large enough to simulate the condition of $T = T_0$ far upstream, i.e., $x \to -\infty$.[13,15]

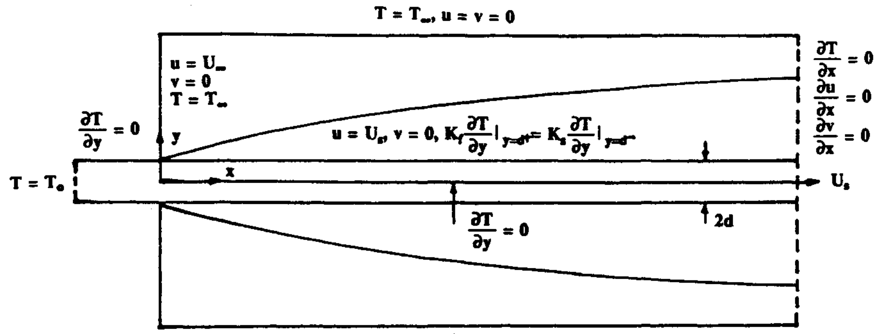

Fig. 2 Typical boundary conditions for a continuously moving plate in a parallel channel flow.

When the temperature gradients are large along the plate length, particularly at small x, the axial diffusion within the plate and the fluid must be taken into account. Therefore, the full governing equations, which are elliptic in nature, must be solved. These full governing equations for the plate and for laminar flow in the fluid, including the transient terms, are

For the fluid

$$\nabla \cdot \bar{v} = 0 \tag{1}$$

$$\frac{\partial \bar{v}}{\partial t} + \bar{v} \cdot \nabla \bar{v} = -\frac{\nabla p}{\rho_f} + \nu \nabla^2 \bar{v} - \bar{g}\beta(T - T_\infty) \tag{2}$$

$$\frac{\partial T}{\partial t} + \bar{v} \cdot \nabla T = \alpha_f \nabla^2 T \tag{3}$$

For the plate

$$\rho_s C_s \left(\frac{\partial T}{\partial t} + U_s \frac{\partial T}{\partial x}\right) = K_s \nabla_s^2 T \tag{4}$$

where the various symbols are defined in the Nomenclature.

The boundary conditions on u, v, and T arise from physical considerations and are shown in Fig. 2. These are due to the no-slip conditions at the moving surface and at the channel wall, temperature and heat flux continuity between the fluid and the solid at the surface of the plate, and symmetry about the x axis, which is taken at the midplane of the plate. When buoyancy effects are included, the symmetry condition at the axis of the plate is applicable only when the plate is moving vertically or when one surface is insulated, with this surface being taken as the x-coordinate axis. In the horizontal case, the problem is not symmetric about the x axis, since the buoyancy forces are directed away from the plate on the upper side and toward the plate on the lower side. We have considered only the upper half of the domain and have applied the condition of symmetry at the midplane of the plate. This assumption is applicable when the bottom surface of the plate is insulated, with the x axis being taken as this surface, or when the buoyancy forces are relatively small.

Additional boundary conditions downstream, in x, are needed because of the elliptic nature of the governing equations. The gradients of the velocity and the temperature with respect to x may be set equal to 0 at the exit, assuming fully developed flow and temperature fields, as has often been done in the literature. For long channels, these conditions may be applied upstream of the exit, and the location where they are applied may be varied numerically until the final solution is marginally affected by a further change. This implies that the fully developed conditions are attained before the exit. Other downstream conditions, particularly setting the second derivatives in x equal to 0, have also been considered in the literature. This approach is appropriate for short channels, since it provides the desired flexibility at the outflow and since the flow and the thermal field may not become fully developed by the exit. This downstream boundary condition is also used in the present study. However, the simpler developed flow conditions were found to be accurate and computationally efficient for the channel lengths usually encountered in practical situations.

The pressure term in Eq. (2) is eliminated by taking the curl of the equation and, thus, transforming it into the vorticity transport equation, as outlined by Jaluria and Torrance.[18] The ψ and ω are defined as

$$u = \frac{\partial \psi}{\partial y}, \qquad v = -\frac{\partial \psi}{\partial x}, \qquad \omega = \frac{\partial v}{\partial x} - \frac{\partial u}{\partial y} \tag{5}$$

Equations (1–5) are nondimensionalized by employing the following transformations:

$$\begin{gathered} X = x/d, \qquad Y = y/d, \qquad \tau = tU_s/d \\ U = u/U_s, \qquad V = v/U_s, \qquad \Psi = \psi/U_s d \\ \Omega = \omega d/U_s, \qquad \theta = \frac{T - T_\infty}{T_0 - T_\infty} \end{gathered} \tag{6}$$

The dimensionless equations thus obtained, in the vorticity-stream function formulation, for a plate which is moving vertically upward, become

For the fluid

$$\nabla^{*2}\Psi = -\Omega \tag{7}$$

$$\frac{\partial \Omega}{\partial \tau} + \bar{V} \cdot \nabla^* \Omega = \frac{1}{Re}(\nabla^{*2}\Omega) - \frac{Gr}{Re^2}\frac{\partial \theta}{\partial Y} \tag{8}$$

$$\frac{\partial \theta}{\partial \tau} + \bar{V} \cdot \nabla^* \theta = \frac{1}{Re \cdot Pr}(\nabla^{*2}\theta) \tag{9}$$

For a plate moving vertically downward, the sign of the buoyancy term is reversed. For the case of a plate moving horizontally, the thermal buoyancy gives rise to a pressure gradient normal to the plate surface,[16] whereas this pressure gradient is aligned with the plate surface in the vertical case. Therefore, in the horizontal case only Eq. (8) is changed, with the applicable equation being

$$\frac{\partial \Omega}{\partial \tau} + \bar{V} \cdot \nabla^* \Omega = \frac{1}{Re}(\nabla^{*2}\Omega) + \frac{Gr}{Re^2}\frac{\partial \theta}{\partial X} \tag{10}$$

The energy equation for the moving plate is

$$\frac{\partial \theta}{\partial \tau} + \frac{\partial \theta}{\partial X} = \frac{1}{Pe}(\nabla_s^{*2}\theta) \tag{11}$$

The relevant boundary conditions have been discussed earlier and are cast in terms of the stream function and vorticity by using the transformations given by Eq. (5).[13,15]

Only a brief discussion of the mathematical formulation is given here since it is similar to that presented in earlier papers. The main difference lies in the presence of the additional forced flow and the geometry being that of a channel in which the flat plate moves. All the three mechanisms for generating the flow, 1) material motion, 2) thermal buoyancy, and 3) forced flow, are assumed to be in the same direction for the vertical case for simplicity in this study. However, opposing circumstances are also of practical interest and need a similar detailed investigation. Therefore, the present work focuses on the relevant boundary conditions for the aiding circumstance and on the effect of the additional forced flow and of the channel geometry on the resulting thermal transport.

Numerical Scheme

In all the cases considered here, the numerical calculations were carried out for only half of the computational domain. The vorticity transport equation, Eq. (8) or (10), was solved along with the energy equation, Eqs. (9) and (11), in the conservative form[18] for transport in the flow and in the plate, respectively, using the Alternate Direction Implicit (ADI) scheme. The Poisson equation, Eq. (7), for the stream function was solved using the successive-over-relaxation (SOR) method. At each time step, after advancing the solution for Eqs. (8–11) by the ADI scheme, Eq. (7) was solved till the maximum change in Ψ satisfied a specified convergence criterion. The optimum relaxation factor was found to be about 1.75, and convergence was obtained in only a few iterations. Three-point central differences were used to discretize all the terms except the convection terms, for which a second-order upwind differencing method was used.

The truncation error for the ADI method is of the order of $[\Delta X^2, \Delta Y^2, \Delta\tau^2]$ for linear equations. However, this second-order accuracy is not maintained when the governing nonlinear equations are linearized by assuming U and V constant over a time step. In addition, the ADI method requires the boundary values of vorticity at the plate surface. The boundary values are given in terms of the Ψ values calculated at the previous time step. This makes the method unstable for larger time steps although the von Neumann analysis indicates unconditional stability for linear problems.[18] For the present study, the scheme was found to be stable for time step $\Delta\tau \leq 0.005$. However, smaller time steps were considered to ensure that the results obtained are independent of the time step employed. It was also confirmed that the steady-state results were not significantly affected by a variation in the initial conditions.[15]

The temperature θ within the solid is first determined, followed by a calculation of the temperature in the flow, employing the boundary conditions at the plate surface. This boundary condition is derived by using the energy balance over the control volume around a grid point located on the surface of the plate. The upper half of the control volume lies within the fluid and the lower half within the plate. The new value of θ is used in calculating the vorticity in the flow at this time step. The stream function equation, which requires only the values of Ψ on the boundaries and the values of the vorticity at interior points, is then solved to give the new Ψ distribution. These new values of Ψ are used to determine the velocity components U and V and the new boundary conditions for the vorticity at the solid walls. This yields a computational process for calculating the temperature, vorticity, stream function, and velocity components for the mixed convection problem under consideration.

The calculations were terminated when $[\max(\partial\theta/\partial\tau)]$ and $[\max(\partial U/\partial\tau)]$ are less than a chosen small quantity ε indicating steady-state conditions. The effect of this ε on the steady-state results was also studied, to ensure a negligible dependence on the value chosen. Similarly, other numerical parameters, such as grid size, time step, X_b, etc., were varied to ensure that the results are independent of the values chosen. The calculations performed on a CDC Cyber 205 machine required 380K words of computer storage and 0.1 s of CPU time per iteration with vectorization. About 30,000 iterations were needed to obtain convergence to steady state when the fluid medium was water, $Pr = 7.0$. However, when air is employed as the fluid, i.e., $Pr = 0.7$, about three times the number of iterations were needed to reach steady state. The total CPU time was about 0.9 h for a typical computational run, in which the plate moves in a water flow channel.

Numerical Results and Discussion

An analytical and numerical model has been developed to investigate the conjugate transport from a heated plate moving in a channel, with the forced flow in the same direction as the motion of the plate. Numerical solutions to Eqs. (7–11) were obtained over wide ranges of the governing parameters Pe, Gr, Pr, K_f/K_s, U_∞/U_s, and Re. Also, solutions were obtained for two geometries, one when the heated plate moves vertically upward and the other when it moves horizontally in the parallel channel flow. Some of the typical results are presented here. The channel width H/d was varied, and the effect of the channel width on the flowfield, as well as on the heat transfer rate, were studied by Kang and Jaluria.[19] It was found that an increase in the channel width increases the heat transfer from the plate to the forced flow. However, the results are not significantly affected by a further increase in the channel width at values of H/d larger than about 8.5. Thus, H/d is fixed at 8.5 in the present study, in order to approximate a wide channel, which is often employed in the cooling of heated moving materials undergoing thermal processing. Water cooling in extrusion is an example of this circumstance.

There are no experimental results available in the literature for the problem under consideration. The only results that may be employed for the validation of the mathematical and numerical model are those obtained for a heated plate or cylinder moving in a quiescent isothermal fluid, as studied by Tsou et al.,[8] Chida and Katto,[11] and Karwe and Jaluria.[14] Figure 3 shows a comparison between experimental and numerical results for a heated plate moving in a stationary fluid medium. Clearly, a fairly good agreement between the two is observed, lending support to the numerical scheme for transport from a flat plate moving in a stationary fluid. The present numerical approach for the channel flow circumstance is similar to the earlier scheme and, thus, this comparison may be taken as an indication of the accuracy and validity of the numerical results since no experimental results are available for the present problem.

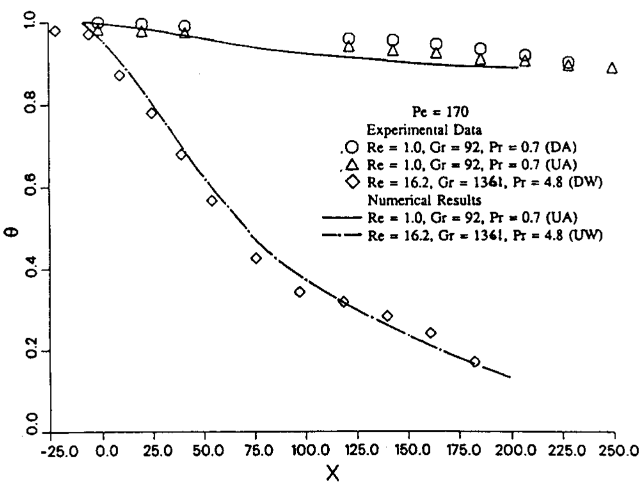

Fig. 3 Comparison between numerical and experimental results on the midplane temperature at steady state for air (A) and water (W) with the plate moving vertically upward (U) or downward (D) in the experiments, for an extensive, quiescent medium.

The computed flow and thermal fields, in terms of the velocity vectors and the isotherms, are shown in Fig. 4 for two geometric circumstances. In the first case a heated aluminum plate moves vertically upward in a water channel flow, and in the second case the heated aluminum plate moves horizontally. A uniform flow at velocity U_∞ is taken at $X = 0$. The flowfield is seen to develop very rapidly downstream and to yield velocity levels higher than U_∞ for both the configurations. The isotherms are also similar in the two cases. However, the thermal boundary layer is thicker when the plate moves horizontally than when it moves vertically upward. This is obviously due to the buoyancy force which is normal to the surface in the horizontal case and is parallel in the vertical configuration. This also implies a lower heat transfer rate in the former case. The temperature distribution across the plate width is seen to be fairly uniform due to the high thermal conductivity of the material. It is also seen that the temperature at $X = 0$ is a strong function of the heat transfer occurring from the material downstream of the slot, indicating the effect of axial conduction within the plate.

The calculated profiles of the U velocity component and temperature θ at various downstream locations X are shown in Fig. 5 for a plate moving vertically upward, and in Fig. 6 for a plate moving horizontally. Since Y is measured from the midplane of the plate, $Y = 1.0$ represents the moving surface. Therefore, the dimensionless velocity $U = u/U_s$ is uniform at 1.0 from $Y = 0.0–1.0$. While the velocity profiles in the vertical configurations shift towards the plate surface as the flow moves downstream, those in the horizontal configuration approach the fully developed flow circumstance. This is expected from the effect of thermal buoyancy which is much stronger in the vertical circumstance. Because of thermal buoyancy, a larger velocity is obtained near the plate in the vertical circumstance than that in the horizontal case. This larger U velocity component in the fluid near the heated plate gives rise to greater energy transfer rate from the plate to the fluid. This results in a lower temperature level within the plate for the vertical configuration than that for the horizontal case, as seen from a comparison of Figs. 5b and 6b. It is also found

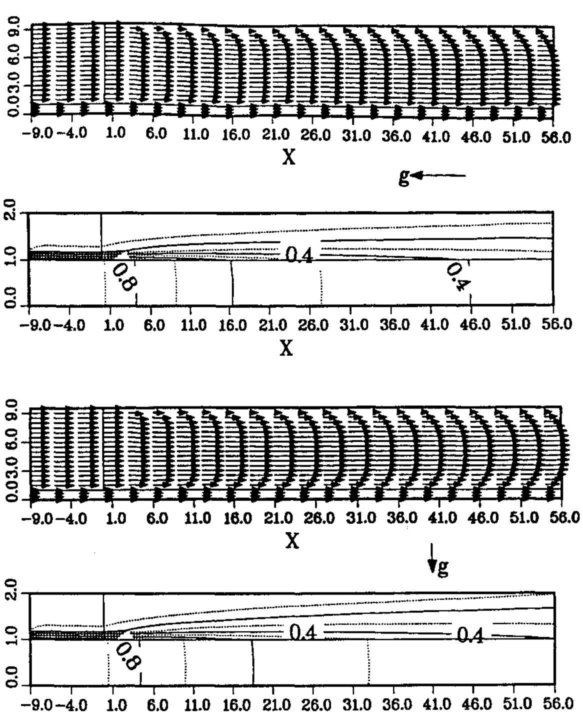

Fig. 4 Calculated velocity field and isotherms for the typical circumstance of an aluminum plate moving in the water flow in a channel: $Pr = 7.0$, $Pe = 0.29$, $Gr = 750$, $Re = 25$, $U_\infty/U_s = 2.0$, $K_f/K_s = 0.0029$: a) vertical configuration and b) horizontal configuration.

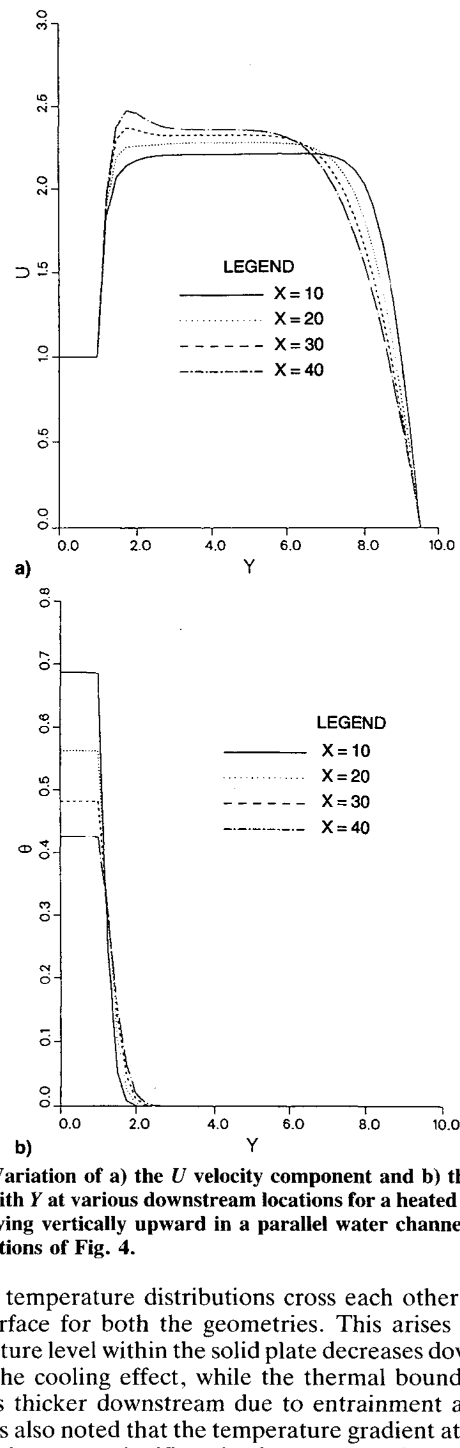

Fig. 5 Variation of a) the U velocity component and b) the temperature θ with Y at various downstream locations for a heated aluminum plate moving vertically upward in a parallel water channel flow, for the conditions of Fig. 4.

that the temperature distributions cross each other near the plate surface for both the geometries. This arises since the temperature level within the solid plate decreases downstream due to the cooling effect, while the thermal boundary layer becomes thicker downstream due to entrainment and diffusion. It is also noted that the temperature gradient at the plate surface decreases significantly downstream due to this increasing boundary-layer thickness, while the surface temperature does not change much. These trends will obviously affect the heat transfer mechanisms, as discussed below.

Figure 7 shows the downstream variation of Nu for the two different geometries. Nu is defined as

$$Nu = \frac{h \cdot d}{K_f} = \left[\left(-\frac{\partial\theta}{\partial Y}\right)_{Y=1.0} \Big/ (\theta)_{Y=1.0}\right] \tag{12}$$

where h is given by

$$h = \left(-K_f \frac{\partial T}{\partial Y}\bigg|_f\right)_{Y=1.0} \Big/ (T_s - T_\infty) \tag{13}$$

It is found that, as expected, the heat transfer rate in the vertical circumstance is higher than in the horizontal one. The

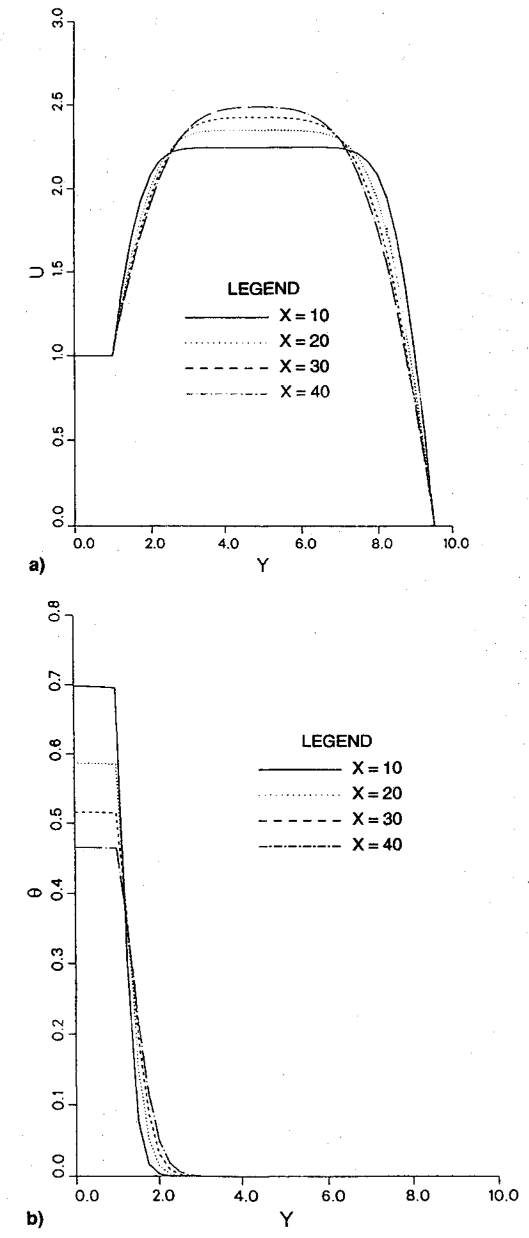

Fig. 6 Variation of a) the U velocity component and b) the temperature θ with Y at various downstream locations for a heated aluminum plate moving horizontally in a parallel water channel flow, for the conditions in Fig. 4.

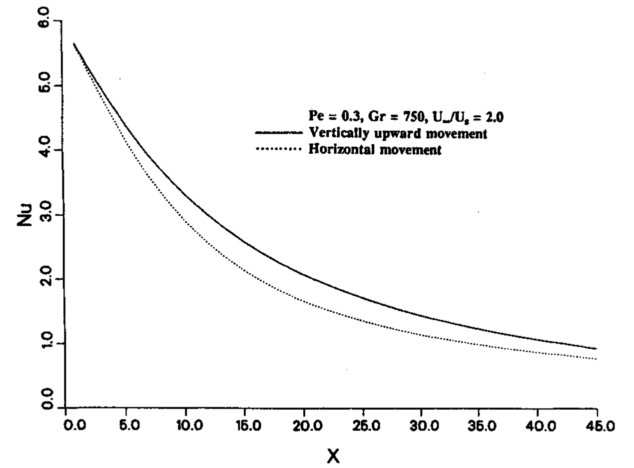

Fig. 7 Variation of Nu for the two configurations of a heated aluminum plate moving in a parallel water channel flow.

difference between the two local Nusselt number distributions initially increases with the downstream distance X, and then decreases far downstream. At small X, the fluid near the plate has not heated up. The fluid gets heated gradually downstream to give rise to an aiding buoyancy circumstance. This increases the heat transfer rate for the vertical orientation, above that for the horizontal one. However, far downstream, the plate cools down and, therefore, the buoyancy effects due to the geometric configuration become weak, leading to the observed smaller difference in the local Nusselt number. Similar trends were observed at other values of the governing parameters.

Gr is varied at given plate speed U_s and forced flow velocity in the channel in Figs. 8 and 9, for a heated aluminum plate moving upward in water flow in a channel. The dimensionless velocity $U = u/U_s$ is uniform at 1.0 from $Y = 0.0$ to 1.0 and then rises sharply, followed by a rapid drop, at higher values of Gr. At $Gr = 5000$, the maximum dimensionless velocity is about 3.7, which is 1.85 times the channel inlet velocity U_∞, since $U_\infty/U_s = 2.0$ in this case. A fairly uniform velocity is seen across much of the channel, and then U drops to 0 at the far wall of the channel, due to the no-slip conditions there. It is interesting to note that the velocity U at higher Gr becomes smaller than that at smaller Gr far from the plate. This

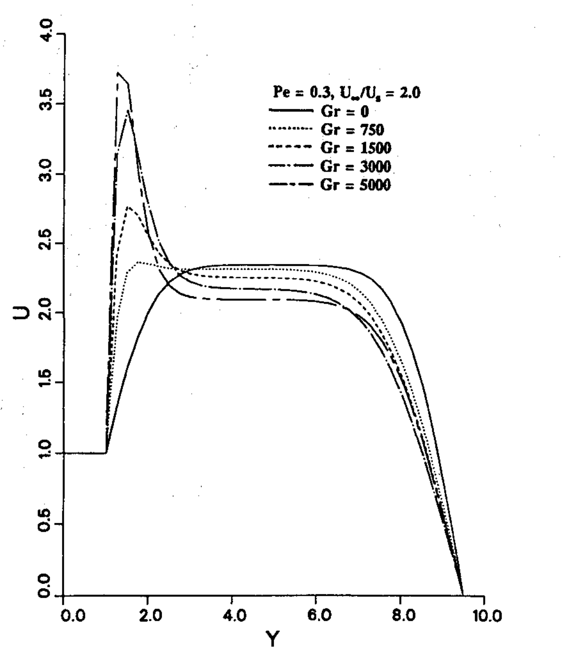

Fig. 8 Computed velocity at $X = 30$ for various values of Gr for a plate moving vertically upward in a channel with the forced flow in the direction of motion at $Re = 25$, $Pr = 7.0$, $Pe = 0.3$, $K_f/K_s = 0.0029$, and $U_\infty/U_s = 2.0$.

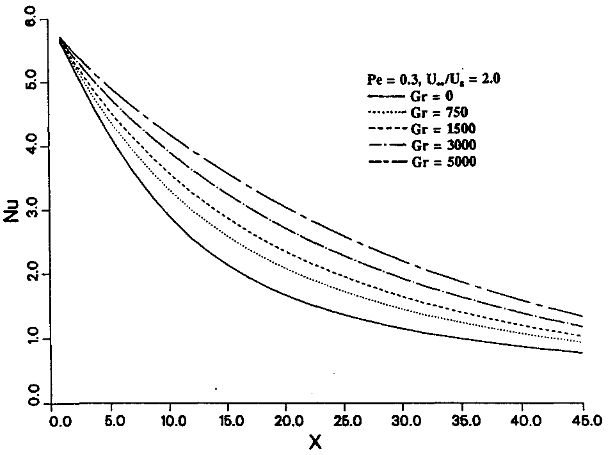

Fig. 9 Effect of Gr on Nu at $Pe = 0.3$ and $U_\infty/U_s = 2.0$.

is clearly because mass conservation must be satisfied by the flow in the channel. It is also found that the strongest buoyancy effect arises near the plate surface since the temperature level of the fluid is the highest in this region.

As seen in Fig. 9, the heat transfer rate increases as the Grashof number increases. It is found that, as expected, the flow is more vigorous in the flow region close to the plate surface where the buoyancy force is stronger. This results in a higher heat transfer rate from the plate to the fluid, leading to larger Nusselt numbers as the Grashof number increases, for fixed plate speed and forced flow velocity in channel.

Figure 10 shows the velocity distributions at $X = 30$ for various U_∞, at a given U_s and Gr, when a heated aluminum plate moves vertically upward. It is seen that, as expected, the buoyancy effect is more significant when the U_∞ is small. At $U_\infty/U_s = 0.5$, the maximum U velocity is about 2.0, which is 4.0 times U_∞. When U_∞/U_s is larger than 5.0, the buoyancy effect is negligible due to the strong forced flow, and this leads to a uniform velocity distribution across the channel except in the regions near the plate surface and near the channel wall, at which the no-slip conditions are applied. The corresponding heat transfer results are shown in Fig. 11, where the downstream variation of Nu is plotted for several values

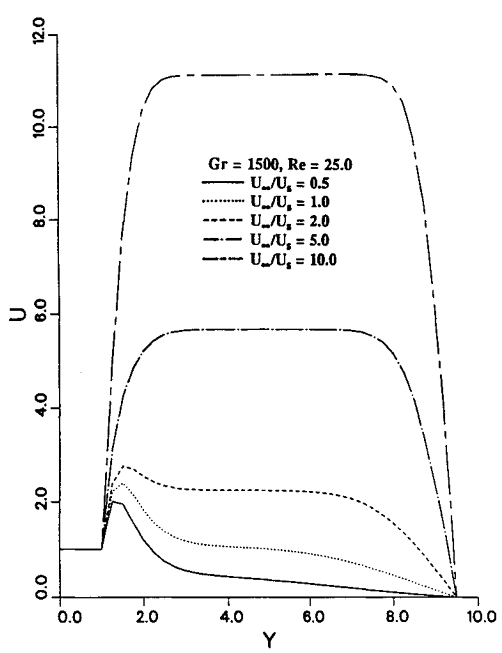

Fig. 10 **Computed velocity distributions at $X = 30$ for various inlet velocities, U_∞/U_s, of the parallel channel flow, when a heated aluminum plate moves vertically upward.**

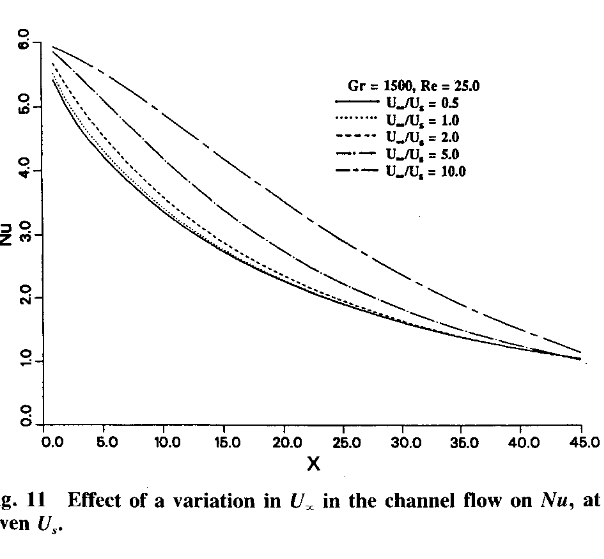

Fig. 11 **Effect of a variation in U_∞ in the channel flow on Nu, at a given U_s.**

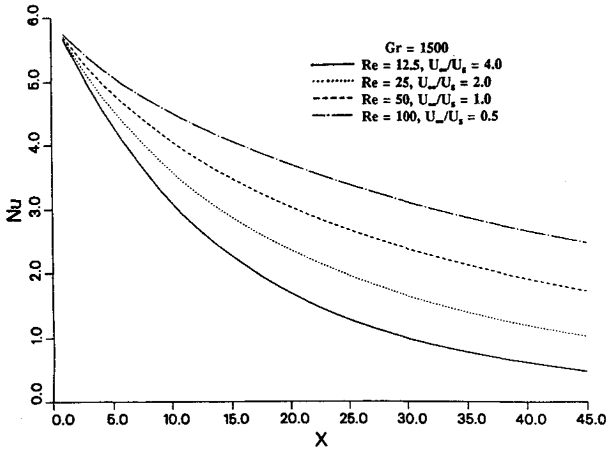

Fig. 12 **Effect of a variation in U_s on Nu at given U_∞ in the channel.**

of the forced flow velocity U_∞/U_s. An increase in this parameter increases the Nusselt number. Thus, the addition of an externally induced forced flow velocity may be employed to enhance the heat transfer rate from the moving plate. This figure also gives the quantitative dependence of Nu on U_∞/U_s, allowing one to estimate the forced flow needed for a desired increase in heat transfer rate.

It is well known that $Ri = Gr/Re^2$, plays a key role in establishing different convection regimes of the flow, such as the natural convection dominated regime at high values of Ri and a forced convection dominated regime at low values of Ri. Thus, Ri indicates the relative intensity of the buoyancy effect in the flow, compared to the viscous effect.[16] When a plate moves through a quiescent medium, U_s and the plate thickness are the relevant characteristic quantities. However, when a plate moves in the forced flow in a channel, U_∞ and H also arise as characteristic quantities. In the present analysis, the plate speed and the plate thickness are employed as the scales to obtain the dimensionless variables. These quantities are consistent when the results for a plate moving in channel flow are compared with those for plate moving in a quiescent extensive medium. However, when the forced flow velocity in the channel U_∞ is changed, Ri based on U_s is meaningless. In Fig. 10, Ri, based on the plate speed, is kept at 2.4 for all cases. But the buoyancy effects in the flow vary with the value of U_∞. Therefore, another Richardson number must be introduced to explain this phenomenon. This Richardson number is based on the additional U_∞, i.e., $Ri_\infty = Gr/Re_\infty^2$, where $Re_\infty = U_\infty d/\nu$. This new Ri_∞, based on the forced flow, varies from 1.2 at $U_\infty/U_s = 0.5$ to 24.0 at $U_\infty/U_s = 10.0$.

Figure 12 shows the local Nusselt number variation with the downstream distance X for several values of U_s at fixed Gr and U_∞. Even though the velocity of the additional U_∞ is kept constant, the ratio of the velocity U_∞/U_s varies because the plate speed U_s changes for each case. An increase in the plate speed increases the heat transfer rate substantially, as expected. At a higher velocity of the plate, the induced flow in the fluid increases, which in turn, results in an increase in the rate of heat removal. As seen earlier, heat transfer from the heated plate to the fluid strongly depends on the local velocity near the plate surface. Thus, the flow induced by the moving plate significantly affects the heat transfer rate. It is also interesting to note that Ri_∞ is kept constant at 0.6 while Ri varies from 9.6 at $Re = 12.5$ to 0.15 at $Re = 100$. It is found by combining Figs. 9 and 12 that larger values of heat transfer rates are obtained at higher values of the plate speed, for a fixed Gr, and at higher values of Gr, with a fixed plate speed. Therefore, Ri cannot be an independent variable in this problem, and Gr and Re are employed as separate, independent variables instead.

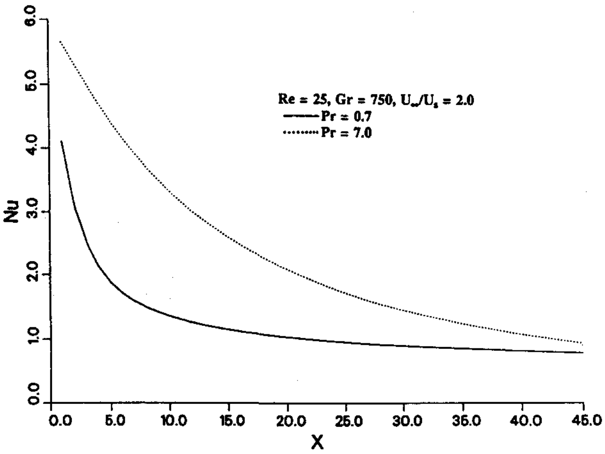

Fig. 13 Downstream variation of *Nu* for a heated aluminum plate moving vertically upward in a water flow and in an airflow.

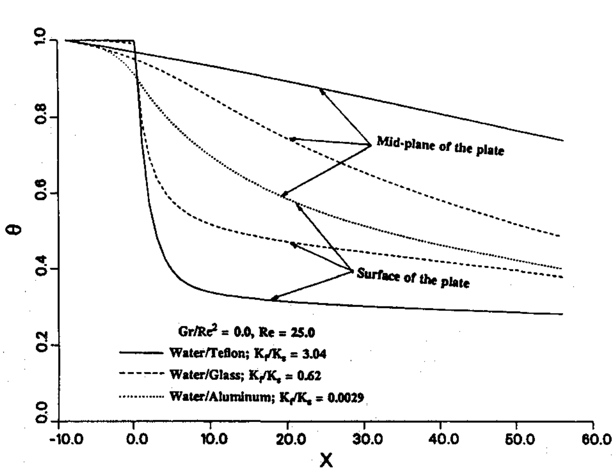

Fig. 14 Downstream variation of the midplane and surface temperatures for various materials of the plate moving in water.

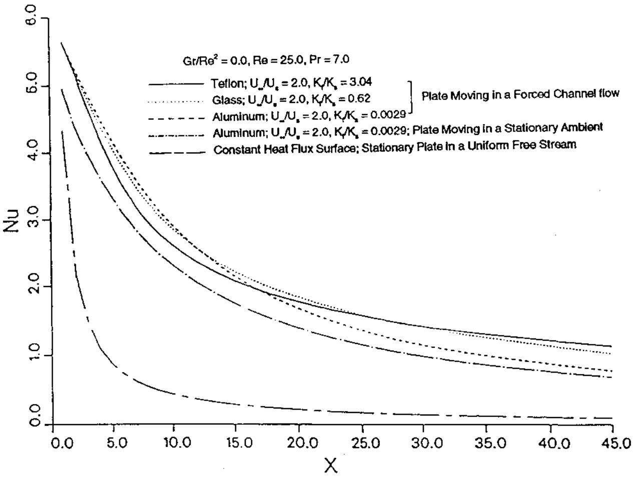

Fig. 15 Effect of the plate material on *Nu*. Also shown are the local Nusselt numbers for a stationary plate in a uniform freestream and for a plate moving in a stationary fluid.

Figure 13 shows the effect of the chosen fluid on the heat transfer rate for a given plate material, taken as aluminum here. It was observed that the velocity distributions were similar to one another, but the flow with air, $Pr = 0.7$, is more vigorous than that with water, $Pr = 7.0$, as the fluid medium. This is expected because the temperature levels are higher in air leading to larger buoyancy effects. However, the thermal boundary-layer thickness is much smaller for water and the thermal conductivity is larger. Therefore, the local Nusselt number is seen to be higher for water than for air, as expected. The biggest difference between the two local Nusselt number distributions arises at about $X = 5.0$ in Fig. 13. It is seen that the heat transfer rate for water is about 2.4 times that for the air at this location. A comparison was also made between the local h from the similarity solution for flow induced by a continuously moving plate through an otherwise quiescent medium, and that from the results for uniform flow over a semi-infinite, stationary plate. The coefficient h with water is 3.9 times that with air in the former case, and 2.1 times in the latter case. Thus, the value of 2.4 obtained for the present problem lies between the two values obtained for flow over a stationary plate and for the flow induced by the plate moving in an isothermal, quiescent, ambient fluid.

Different materials of the plate may also be considered with a given fluid medium. Figure 14 shows plates of three different materials, Teflon®, glass, and aluminum, moving in a water channel. It is interesting to note that the upstream penetration of thermal effects is small for Teflon, due to its low thermal conductivity. Also, the surface temperature decreases very rapidly downstream because of the small diffusion in the radial and axial directions. This gives rise to the largest difference between the surface and the midplane temperatures for Teflon. Glass is the intermediate material, in terms of the thermal conductivity, and the observed trends lie between those for Teflon and aluminum. The downstream penetration of the thermal field is the largest for aluminum and the temperature difference across the plate is the smallest, as expected from the large thermal conductivity of the material. These trends reinforce the observations and discussions presented earlier on the resulting heat transfer mechanisms.

Figure 15 shows the effect of the material of the plate on the local Nusselt number variation along the downstream distance X. The trends indicating the effect of the material are very similar to those for a plate moving in a quiescent medium.[15] The local Nusselt number variation for a plate moving in a channel flow is also compared with that for a plate moving in a stationary medium, and with that from the analytical solution for an externally driven flow over a stationary semi-infinite plate in this figure. Here, the analytical solution is obtained for a uniform heat flux plate. While the temperature distributions in the transverse direction are similar in all these cases, the velocity distributions are different. It is seen that the local Nusselt number is much higher for a moving plate than that for a stationary one. This means that, for a given fluid and plate velocity U_s, the moving plate loses energy to the flow at a faster rate. This result can be explained in terms of the transverse v. This component of velocity is negative for the moving plate and positive for the stationary plate. Therefore, in the case of a moving plate, fluid at the lower ambient temperature is brought towards the plate. This enhances the rate of heat removal from the plate surface. It is also seen in this figure that the heat transfer rate for a plate moving in a channel flow is higher than that for a plate moving in a quiescent fluid medium, since the additional forced flow aids the thermal transport.

These results are valuable in the evaluation of the effects of the important physical variables in the problem such as the temperature level, plate speed, flow orientation, ambient flow velocity, and material and fluid properties on the overall transport process. They are useful, for instance, in determining the dimensions of the system needed to bring the material temperature to a desired level. In addition, the variation in material characteristics arising due to temperature gradients within the plate can be controlled, as well as estimated. The results presented here are of interest and importance in the design of practical systems related to processes such as rolling, drawing, and extrusion.

Conclusions

A detailed numerical study of the conjugate transport from a continuously moving heated plate in a parallel channel flow has been carried out. Two geometric configurations are considered. These are the plate moving horizontally or vertically upward. The basic approach applies to many processes ranging from crystal growing and continuous casting to extrusion, rolling, wire drawing, and fiber drawing. The numerical results obtained here indicate the temperature decay in the material as it moves away from the die or furnace in such manufacturing processes. The resulting heat transfer and the characteristics of the flow are determined. It is found that the thermal boundary layer is thinner when a plate is moving vertically upward than when it is moving horizontally, as expected from the alignment of the buoyancy force with the motion in the former case. The velocity profiles in the vertical configuration are shifted toward the plate surface as the flow moves downstream, whereas those in the horizontal configuration approach the fully developed channel flow downstream. Also, the heat transfer rate in the vertical circumstance is higher than that in the horizontal orientation.

A strong forced flow is found to lead to an essentially uniform velocity distribution across much of the channel. It is also found that the rate of heat removal from the heated plate largely depends on the local velocity near the plate surface, rather than the velocity level over much of the cross section of the channel. Thus, the plate speed affects the heat transfer rate significantly. The heat transfer rate for a given fluid, e.g., water or air, for a moving plate in a channel is larger than that for flow induced by a plate moving through a quiescent medium. The upstream penetration of thermal effects, as well as the difference between the surface and the midplane temperatures, are greatly affected by the plate material, particularly the thermal conductivity of the material.

Acknowledgments

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References

[1]Tadmor, Z., and Klein, I., *Engineering Principles of Plasticating Extrusion*, Polymer Science and Engineering Series, Van Nostrand Reinhold Co., New York, 1970.

[2]Fisher, E. G., *Extrusion of Plastics*, Wiley, New York, 1976.

[3]Atlan, T., Oh, S., and Gegel, H., *Metal Forming Fundamentals and Applications*, American Society of Metals, Metals Park, OH, 1979.

[4]Abdelhafez, T. A., "Skin Friction and Heat Transfer on a Continuous Flat Surface Moving in a Parallel Free Stream," *International Journal of Heat and Mass Transfer*, Vol. 28, No. 6, 1985, pp. 1234–1237.

[5]Rhodes, C. A., and Chen, C. C., "Thermal Radiation in Laminar Boundary Layer on Continuous Moving Surfaces," *Journal of Heat Transfer*, Vol. 96, Feb. 1974, pp. 32–36.

[6]Zumbrunnen, D. A., Viskanta, R., and Incropera, F. P., "The Effect of Surface Motion on Forced Convection Film Boiling Heat Transfer," *Journal of Heat Transfer*, Vol. 111, Aug. 1989, pp. 760–766.

[7]Sakiadis, B. C., "Boundary-Layer Behavior on Continuous Solid Surfaces: I. Boundary Layer Equations for Two-Dimensional and Axisymmetric Flow," *AIChE Journal*, Vol. 7, No. 1, 1961, pp. 26–28.

[8]Tsou, F. K., Sparrow, E. M., and Goldstein, R. J., "Flow and Heat Transfer in the Boundary Layer on a Continuous Moving Surface," *International Journal of Heat and Mass Transfer*, Vol. 10, No. 2, 1967, pp. 219–235.

[9]Griffin, J. F., and Thorne, J. L., "On Thermal Boundary Layer Growth on Continuous Moving Belts," *AIChE Journal*, Vol. 13, No. 6, 1967, pp. 1210, 1211.

[10]Moutsoglou, A., and Chen, T. S., "Buoyancy Effects in Boundary Layers on Inclined, Continuous, Moving Sheets," *Journal of Heat Transfer*, Vol. 102, May 1980, pp. 371, 372.

[11]Chida, K., and Katto, Y., "Conjugate Heat Transfer of Continuously Moving Surfaces," *International Journal of Heat and Mass Transfer*, Vol. 19, No. 5, 1976, pp. 461–470.

[12]Karwe, M. V., and Jaluria, Y., "Fluid Flow and Mixed Convection Transport from a Plate in Rolling and Extrusion Processes," *Journal of Heat Transfer*, Vol. 110, Aug. 1988, pp. 655–661.

[13]Karwe, M. V., and Jaluria, Y., "Numerical Simulation of Thermal Transport Associated with a Continuously Moving Flat Sheet in Materials Processing," *Journal of Heat Transfer*, Vol. 113, Aug. 1991, pp. 612–619.

[14]Karwe, M. V., and Jaluria, Y., "Experimental Investigation of Thermal Transport from a Heated Moving Plate," *International Journal of Heat and Mass Transfer*, Vol. 35, No. 2, 1992, pp. 493–511.

[15]Kang, B. H., Jaluria, Y., and Karwe, M. V., "Numerical Simulation of Conjugate Transport Form a Continuous Moving Plate in Materials Processing," *Numerical Heat Transfer*, Vol. 19, No. 2, 1991, pp. 151–176.

[16]Jaluria, Y., *Natural Convection Heat and Mass Transfer*, Pergamon, New York, 1980.

[17]Paek, U. C., and Schroeder, C. M., "Forced Convective Cooling of Optical Fiber in High-Speed Coating," *Journal of Applied Physics*, Vol. 50, No. 10, 1979, pp. 6144–6148.

[18]Jaluria, Y., and Torrance, K. E., *Computational Heat Transfer*, Taylor & Francis, Washington, DC, 1986.

[19]Kang, B. H., and Jaluria, Y., "Numerical Study of the Fluid Flow and Heat Transfer Due to a Heated Plate Moving in a Uniform Forced Flow," *Numerical Heat Transfer*, Vol. 22, No. 2, 1992, pp. 143–165.